Supersymmetric Standard Model from String Theory

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Abstract

N=1, D=4 Superstring possessing a $SO(6) \otimes SO(5)$ symmetric action and with the same gauge symmetry obtained from the zero mass spectrum of vector meson as well, is constructed from the bosonic string in twenty six dimensions. Without breaking supersymmetry, the gauge symmetry of the model descends to the supersymmetric standard model of the electroweak scale in four dimension. It is proved that there can be three generations in the model.

PACS numbers: 11.25-w, 11.30Pb, 12.60.Jv

The model begins from the Nambu-Goto [1, 2] bosonic string theory in the world sheet (σ, τ) which makes sense in 26 dimensions. The reason is easy to see. The string action in twenty six dimension is

$$S_B = -\frac{1}{2\pi} \int d^2\sigma \left(\partial_\alpha X^\mu \partial^\alpha X_\mu\right), \quad \mu = 0, 1, 2, ..., 25. \tag{1}$$

where $\partial_\alpha = (\partial_\sigma, \partial_\tau)$. The central charge for bosons is easily found by using the general expression for the two energy momentum tensors at two world sheet points z and ω

$$2 < T(z)T(\omega) >= \frac{C}{(z-\omega)^4} + ... \tag{2}$$

The coefficient of the most divergent term as C in equation (2) is the central charge. For free bosons, the central charge is

$$C_B = \delta^\mu_\mu \tag{3}$$

For equation (1) i.e. the action S_B has a central charge δ^μ_μ=26. So the action (1) is not anomaly free. One adds to equation (1) of the action of the conformal ghosts (c^+, b_{++})

$$S_{FP} = \frac{1}{\pi} \int \left(c^+ \partial_- b_{++} + c^- \partial_+ c_{--}\right) d^2\sigma \tag{4}$$

This action (4) has a central charge -26, independent of the dimensionality of the string. To have an anomaly free string theory, the central charge of the conformal ghosts should be able to cancel only when the C=δ^μ_μ=D=26. The string is physical only in D=26 dimension with the total central charge zero. Using Mandelstam's [3] proof of equivalence between one boson to two fermionic modes in the infinite volume limit in (1+1) dimensional quantum field theory, one can rewrite the action as the sum of the four bosonic coordinates X^μ of SO(3,1) and forty four fermions having internal symmetry SO(44). If this is anomaly free, this is also true in finite intervals or a circle. Noting that the Majorana fermions can be in bosonic representation of the Lorentz group SO(3,1), the forty four fermions are grouped into eleven Lorentz vectors of SO(3,1) which look as a commuting internal symmetry group when viewed from the other internal quantum number space. The action is now

$$S_{FB} = -\frac{1}{2\pi} \int d^2\sigma \left[\partial^\alpha X^\mu(\sigma,\tau)\, \partial_\alpha X_\mu(\sigma,\tau) - i \sum_{j=1}^{11} \bar{\psi}^{\mu,j} \rho^\alpha \partial_\alpha \psi_{\mu,j}\right]. \tag{5}$$

and is anomaly free with S_{FP} of equation (4). The upper indices j,k refer to a row and that of lower to a column and

$$\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{6}$$

and

$$\rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \tag{7}$$

Dropping indices

$$\bar{\psi} = \psi^\dagger \rho^0. \tag{8}$$

ρ^α's are imaginary so the Dirac operators $\rho^\alpha \partial_\alpha$ is real and this representation of Dirac algebra, the components of the world sheet spinor $\psi^{\mu,j}$ are real and are Majorana spinors.

One has introduced an anticommuting field $\psi^{\mu,j}$ that transforms as vectors - a bosonic representation of SO(3,1). ψ^μ_A maps bosons to bosons and fermions to fermions in the space time sense. There is no clash with spin statistics theorem. Action (5) is a two dimensional field theory, not a field theory is space time. ψ^μ_A transforms as a spinor under the transformation of the two dimensional world sheet. The Lorentz group SO(3,1) is merely an internal symmetry group as viewed in the world sheet. This is discussed in reference [4].

The central charge of the the free fermions in action (5) as deduced by calculation from equation (2) is

$$C_F = \frac{1}{2}\delta^\mu_\mu \delta^j_j, \tag{9}$$

so that the total central charge of equation (5) is

$$C_{SB} = \delta^\mu_\mu + \delta^\mu_\mu \delta^j_j = 4 + \frac{1}{2} \times 4 \times 11 = 26 \tag{10}$$

The total central charge is zero with S_{FP}. As a check of equation (10), the central charge of a ten dimensional superstring is $10+\frac{1}{2} \times 10 \times 1 = 15$. So this formula is correct. However, the action (5) is not yet supersymmetric. The eleven $\psi^{\mu,j}_A$ have to be further divided into two species; $\psi^{\mu,j}$, j=1,2,..6 and $\phi^{\mu,k}$, k=7,8..11. For the group of six, the positive and negative parts are $\psi^{\mu,j} = \psi^{(+)\mu,j} + \psi^{(-)\mu,j}$ whereas for the group of five, allowed the freedom of phase of creation operators for Majorana fermions in $\phi^{\mu,k} = \phi^{(+)\mu,k} - \phi^{(-)\mu,k}$. The action is now

$$S = -\frac{1}{2\pi} \int d^2\sigma \left[\partial_\alpha X^\mu \partial^\alpha X_\mu - i\ \bar{\psi}^{\mu,j}\ \rho^\alpha\ \partial_\alpha\ \psi_{\mu,j} + i\ \bar{\phi}^{\mu,k}\ \rho^\alpha\ \partial_\alpha\ \phi_{\mu,k}\right] \tag{11}$$

Besides SO(3,1), the action (11) is invariant under $SO(6) \otimes SO(5)$. It is also invariant under the supersymmetric transformation

$$\delta X^{\mu} = \bar{\epsilon}\ (e^{j}\psi^{\mu}_{j} - e^{k}\phi^{\mu}_{k}), \tag{12}$$

$$\delta\psi^{\mu,j} = -ie^{j}\rho^{\alpha}\partial_{\alpha}X^{\mu}\ \epsilon \tag{13}$$

$$\delta\phi^{\mu,k} = ie^{k}\rho^{\alpha}\partial_{\alpha}X^{\mu}\ \epsilon. \tag{14}$$

ϵ is a constant anticommuting spinor. e^{j} and e^{k} are taken as eleven numbers of a row matrix with $e^{j}e_{j}$=6 and $e^{k}e_{k}$=5. They look like e^{1}=(1,0,0,0,0,0,0,0,0,0,0) and e^{7}=(0,0,0,0,0,0,1,0,0,0,0).

There is wide mismatch between the fermionic and bosonic modes in the action (11). To investgate these disturbing features, we find that the commutators of two successive supersymmetric transformations lead to a translation with the coefficients $a^{\alpha} = 2\ i\ \bar{\epsilon}^{1}\rho^{\alpha}\epsilon_{2}$ provided the internal symmetry indices (SO(11) in equation (5) or SO(6)⊗SO(5) in equation(11)) satisfy.

$$\psi^{\mu}_{j} = e_{j}\Psi^{\mu}, \qquad \phi^{\mu}_{k} = e_{k}\Psi^{\mu}, \tag{15}$$

These are the two key equations. They also state that of the eleven sites (j,k), the super fermionic partner Ψ^{μ} is found in one site only. On jth or kth site, it emits or absorbs quanta as found by quantising $\psi^{\mu,j}$ or $\phi^{\mu,k}$ respectively prescribed by the action (11). The alternative auxilliary fields are not needed. The Ψ^{μ} is given by

$$\Psi^{\mu} = e^{j}\psi^{\mu}_{j} - e^{k}\phi^{\mu}_{k}. \tag{16}$$

It is easy to verify that there is no e^{j} or e^{k} and

$$\delta X^{\mu} = \bar{\epsilon}\Psi^{\mu}, \qquad \delta\Psi^{\mu} = -i\ \epsilon\ \rho^{\alpha}\ \partial_{\alpha}\ X^{\mu} \tag{17}$$

and

$$[\delta_1,\delta_2]X^{\mu} = a^{\alpha}\partial_{\alpha}X^{\mu}, \qquad [\delta_1,\delta_2]\Psi^{\mu} = a^{\alpha}\partial_{\alpha}\Psi^{\mu} \tag{18}$$

We immediately obtain a Nambu-Goto superstring in four dimensions from the action (11) using equation (15). The action is

$$S = -\frac{1}{2\pi}\int d^{2}\sigma\,(\partial_{\alpha}X^{\mu}\partial^{\alpha}X_{\mu} - \ i\Psi^{\mu}\rho^{\alpha}\partial_{\alpha}\Psi_{\mu}) \tag{19}$$

Thus the action(11) is truely supersymmetric. Quantising this action in well written down procedure, the particle spectrum is very rich unlike quantising (19). We also need the internal symmetry $SO(6)\otimes SO(5)$ of the action (11).

From equations (17) and (18), it follows that the superpartner of X^μ is Ψ^μ. Introducing another supersymmetric pair the Zweibein $e^\alpha(\sigma,\tau)$ and the gravitons $\chi_\alpha = \nabla_\alpha\epsilon$, the local 2-d supersymmetric action, first written down by Brink, Di Vecchia, Howe, Deser and Zumino [5, 6] is

$$S = -\frac{1}{2\pi}\int d^2\sigma \; e \; \left[h^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X_\mu - i\bar{\Psi}^\mu\rho^\alpha\partial_\alpha\bar{\Psi}_\mu + 2\bar{\chi}_\alpha\rho^\beta\rho^\alpha\Psi^\mu\partial_\beta\chi^\mu + \frac{1}{2}\bar{\Psi}^\mu\Psi_\mu\bar{\chi}_\beta\rho^\beta\rho^\alpha\chi_\alpha\right] \tag{20}$$

This action has several invariances. For detailed discussions, see reference [4]. Varying the field and Zweibein, vanishing of the Noether current J^α and the energy momentum tensor $T_{\alpha\beta}$ is derived.

$$J_\alpha = \frac{\pi}{2e}\frac{\delta S}{\delta\chi^\alpha} = \rho^\beta\rho_\alpha\bar{\Psi}^\mu\partial_\beta X_\mu = 0 \tag{21}$$

and

$$T_{\alpha\beta} = \partial_\alpha X^\mu\partial_\beta X_\mu - \frac{i}{2}\bar{\Psi}^\mu\rho_{(\alpha}\partial_{\beta)}\Psi_\mu = 0. \tag{22}$$

In a light cone basis, the vanishing of the light cone components are

$$J_\pm = \partial_\pm X_\mu\Psi^\mu_\pm = 0 \tag{23}$$

and

$$T_{\pm\pm} = \partial_\pm X^\mu\partial_\pm X_\mu + \frac{i}{2}\psi^{\mu j}_\pm\partial_\pm\psi_{\pm\mu,j} - \frac{i}{2}\phi^{\mu k}_\pm\partial_\pm\phi_{\pm\mu,k}, \tag{24}$$

where $\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$.

The central charge of the action (11) is C $= \delta^\mu_\mu + \frac{1}{2}\delta^\mu_\mu(\delta^j_j + \delta^k_k)$=26 also. This is a supersymmetric action and with the conformal ghosts, the anomaly vanishes. But there will be current generator constraints for which we need superconformal ghosts for BRST charge. I shall come back to this problem later in this article.

The action in equation (20) is not space time supersymmetric. However, in the fermionic representation SO(3,1), fermions are Dirac spinors with four components α. We construct Dirac spinor like equation (16) as the sum of component spinors

$$\Theta_\alpha = \sum_{j=1}^{6} e^j\theta_{j\alpha} - \sum_{k=7}^{11} e^k\theta_{k\alpha}. \tag{25}$$

With the usual Dirac matrices Γ^μ, since the identity

$$\Gamma_\mu\psi_{[1}\bar{\psi}_2\Gamma^\mu\psi_{3]} = 0 \tag{26}$$

is satisfied due to the Fierz transformation in four dimension, the Green-Schwarz action [7] for N=1 supersymmetry is

$$S = \frac{1}{2\pi}\int d^2\sigma \left(\sqrt{g}g^{\alpha\beta}\Pi_\alpha\Pi_\beta + 2i\epsilon^{\alpha\beta}\partial_\alpha X^\mu\bar{\Theta}\Gamma_\mu\partial_\beta\Theta\right), \tag{27}$$

where

$$\Pi^\mu_\alpha = \partial_\alpha X^\mu - i\bar{\Theta}\Gamma^\mu\partial_\alpha\Theta. \tag{28}$$

This is the N=1 and D=4 superstring originating from the D=26 bosonic string. It is difficult to quantise this action covariantly. It is better to use NS-R [8, 9] formulation with G.S.O projection [10]. This has been done in reference [11, 14], while quantising each eleven component fermions ψ^μ_j and ϕ^μ_k, there appears eleven additional ghosts, η^{00} being -1. But, there can be similarly 11×2 constraint equations from one equation (23). Since the superpartner Ψ^μ has been identified, we omit further refering to e_j, e_k for easy reading.

To outline briefly, let L_m, G_r and F_m be the Super Virasoro generators of energy, momenta and currents. Let α's denote the quanta of X^μ field, b's and b$'$'s denote quanta of ψ and ϕ fields in NS formulation and d, d$'$ in R formulation. Then,

$$\begin{aligned}
L_m &= \frac{1}{\pi}\int_{-\pi}^{\pi} d\sigma e^{im\sigma}T_{++} \\
&= \frac{1}{2}\sum_{-\infty}^{\infty} :\alpha_{-n}\cdot\alpha_{m+n}: + \frac{1}{2}\sum_{r\in z+\frac{1}{2}}(r+\frac{1}{2}m):(b_{-r}\cdot b_{m+r} - b'_{-r}\cdot b'_{m+r}): \qquad NS \\
&= \frac{1}{2}\sum_{-\infty}^{\infty} :\alpha_{-n}\cdot\alpha_{m+n}: + \frac{1}{2}\sum_{n=-\infty}^{\infty}(n+\frac{1}{2}m):(d_{-n}\cdot d_{m+n} - d'_{-n}\cdot d'_{m+n}): \;, \qquad R
\end{aligned} \tag{29}$$

$$G_r = \frac{\sqrt{2}}{\pi}\int_{-\pi}^{\pi} d\sigma e^{ir\sigma}J_+ = \sum_{n=-\infty}^{\infty}\alpha_{-n}\cdot\left(e^j b_{n+r,j} - e^k b'_{n+r,k}\right), \qquad NS \tag{30}$$

and

$$F_m = \sum_{-\infty}^{\infty}\alpha_{-n}\cdot\left(e^j d_{n+r,j} - e^k d'_{n+r,k}\right). \qquad R \tag{31}$$

and satisfy the super Virasoro algebra with central charge C

$$\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} + \frac{C}{12}(m^3-m)\delta_{m,-n}, && && (32)\\
[L_m, G_r] &= (\frac{1}{2}m - r)G_{m+r}, && NS && (33)\\
\{G_r, G_s\} &= 2L_{s+r} + \frac{C}{3}(r^2 - \frac{1}{4})\delta_{r,-s}, && && (34)\\
[L_m, F_n] &= (\frac{1}{2}m - n)F_{m+n}, && R && (35)\\
\{F_m, F_n\} &= 2L_{m+n} + \frac{C}{3}(m^2-1)\delta_{m,-n}, \quad m \neq 0. && && (36)
\end{aligned}$$

This is also known that the normal ordering constant of L_o is equal to one and we define the physical states as satisfying

$$\begin{aligned}
(L_o - 1)|\phi> &= 0, \quad L_m|\phi>=0, \quad G_r|\phi>=0 \quad for \quad r,m>0, \quad NS \quad Bosonic && (37)\\
L_m|\psi> &= F_m|\psi>=0 \qquad for \quad m>0, \qquad :R \quad Fermionic && (38)\\
(L_o - 1)|\psi>_\alpha &= (F_o^2 - 1)|\psi>_\alpha = 0. && (39)
\end{aligned}$$

So we have

$$(F_o + 1)|\psi_+>_\alpha = 0 \quad and \quad (F_o - 1)|\psi_->_\alpha = 0 \qquad :R \tag{40}$$

These conditions shall make the string model ghost free.

It can be seen in a simple way. Applying L_o condition the state $\alpha^\mu_{-1}|0,k>$ is massless and the L_1 constraint gives the Lorentz condition $k^\mu|0,k>=0$ implying a transverse photon and Gupta Bleuler impose that $\alpha^0_{-1}|\phi>=0$. Applying L_2, L_3 ...., constraints, one obtains $\alpha^0_m|\phi>=0$. Further, since $[\alpha^0_{-1}, G_{r+1}]|\phi>=0$, we have $b^0_{r,j}|\phi>=0$ and $b'^0_{r,k}|\phi>=0$. All the time components are eliminated from Fock space.

The central charge, coming from (11) is 26. This is cancelled by the contribution of the action of the conformal ghosts -26. However, one must have superconformal ghosts which contributes a central charge 11 so that the current generators constraints (38) and (39) are preserved by the BSRT nilpotent charge. We search for a action which is ghost like and and has central charge 11. It is easily found. The light cone part of the action (11) namely

$$S_{l.c} = \frac{i}{2\pi}\int d^2\sigma \sum_{\mu=0}^{3}\left(\phi^{\mu,j}\rho^\alpha\partial_\alpha\phi_{\mu,j} - \phi^{\mu,j}\rho^\alpha\partial_\alpha\phi_{\mu,j}\right) \tag{41}$$

has central charge 11 using equation (9). The superconformal ghost action should be the same in a supplemented ghost space where the fermions behave like bosons and vice versa. A separate action for superconformal ghosts will overcount the central charge by 11.

I examine the possibility of the action $S_F^{l.c}$ contained in equation (11) to be transformable to the action of superconformal ghosts. It is simpler to use $\Psi^\pm = \frac{1}{\sqrt{2}}(\Psi^0 \pm \Psi^3)$ so that the action is

$$S_F^{l.c} = -\frac{i}{\pi}\int d^2\sigma\ \bar{\Psi}^+\rho^\alpha\partial_\alpha\Psi^- \tag{42}$$

One supplement the ghost space by the superconformal ghost space obeying reverse statistics. One can do this by letting $\Psi^\pm(\sigma)$ to be dependent on Grassman variables θ and $\overline{\theta}$ as well. Then the action takes the form

$$S_F^{l.c} = -\frac{i}{\pi}\int d^2\sigma\ \int d\bar{\theta}\int d\theta\ \bar{\Psi}^+(\bar{\theta})\ \rho^\alpha\partial_\alpha\Psi^-(\theta). \tag{43}$$

Expanding the fields as

$$\bar{\Psi}^+(\bar{\theta}) = \cdots + \bar{\theta}\bar{\gamma} + \cdots \tag{44}$$

and

$$+2\ i\ \rho^\alpha\Psi^-(\theta) = \cdots + \theta\beta^\alpha + \cdots \tag{45}$$

The Superconformal ghost action is pulled out as

$$S_F^{l.c} = -\frac{1}{2\pi}\int d^2\sigma\ \bar{\gamma}\partial_\alpha\beta_\alpha \tag{46}$$

The energy momentum tensor is

$$T_{++} = -\frac{1}{4}\gamma\ \partial_+\beta - \frac{3}{4}\beta\partial_+\gamma, \tag{47}$$

and the wave equations are $\partial\gamma = \partial\beta = 0$. This has central charge With 'p' integral for Ramond and half integral for NS, the field quanta $\gamma_p,\ \beta_p$ satisfy the only nonvanishing commutator relation

$$[\gamma_p, \beta_r] = \delta_{p+r}. \tag{48}$$

The $L^{l.c}$-generator is transformed into

$$L_m^{l.c.} = \sum_n\ (n + \frac{1}{2}m)\quad :\beta_{m-n}\gamma_n\ : \tag{49}$$

This is equal to the superconformal ghosts of the normal 10-d superstring.

All that we need to prove that the nilpotency of BSRT charge is that the conformal dimension of γ is '-1/2' and that of β is '3/2' respectively. They can be deduced from equation(49) using equation (48). We now proceed to write the nilpotent BRST charge. The part of the charge which comes from the usual conformal Lie algebra technique is

$$(Q_1)^{NS,R} = \sum (L_{-m}c_m)^{NS,R} - \frac{1}{2}\sum (m-n) : c_{-m}c_{-n}b_{m+n} : \; - \; a\, c_0; \qquad Q_1^2 = 0 \quad for\ a = 1. \tag{50}$$

Using the Graded Lie algebra, we get the additional BRST charge, in a straight forward way, in NS and R,

$$\begin{aligned} Q'_{NS} &= \sum G_{-r}\gamma_r - \sum \gamma_{-r}\gamma_{-s}b_{r+s}, \\ Q'_R &= \sum F_{-m}\gamma_m - \sum \gamma_{-m}\gamma_{-n}b_{n+m}. \end{aligned} \tag{51}$$

As constructed, the BRST charge

$$Q_{BRST} = Q_1 + Q', \tag{52}$$

is such that $Q^2_{BRST} = 0$ in both the NS and the R sector [11]. In proving $\{Q', Q'\} + 2\{Q_1, Q'\} = 0$, we have used the fourier transforms, the wave equations and integration by parts, and to show that

$$\sum\sum r^2 \gamma_r\gamma_s\delta_{r,-s} = \sum_r \sum_s \gamma_r\gamma_s\delta_{r,-s} = 0. \tag{53}$$

Thus the theory is unitary and ghost free.

Interestingly, Polchinski [19] has proved that such actions allow only a very limited possible algebra. They are shown in a table depending on the central charge of the ghosts. The present case belongs to either N=0 with C^{ghost}=-26 or N=1 with C^{ghost}=-15. More interestingly, Polchinski adds that for N=0 and N=1, there can also be additional spin 1 and spin $\frac{1}{2}$ constraints, provided that the supercurrent is neutral under the corresponding symmetry. These larger symmetries are not essentially different. The total central charge of the ghosts allow additional matter but the additional constraints precisely remove the added states, so that they reduce to the N=0 or N=1 theories. In our case, we started out for the action (11) with C_{FP}=-26 i.e. an N=0 supersymmetry. But due to fermionic matter and the current constraints (37) to (40), the superconformal ghost with C^{gh}=11 had had to be added. They were taken out of the action (11) as $S_F^{l.c.}$ which was shown to be the same as superconformal ghost action equation (49). Thus , finally one ends up with N=1 supersymmetry with C^{gh}=-15.

There are no harmful effective tachyons in the model eventhough

$$\alpha' M^2 = -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, ...NS \quad (54)$$

$$\text{and} \quad (55)$$

$$\alpha' M^2 = -1, 0, 1, 2, 3, ...R. \quad (56)$$

The G.S.O. projection eliminates the half integral values. The tachyonic self energy of bosonic sector $< 0|(L_o - 1)^{-1}|0 >$ is cancelled by $- < 0|(F_o + 1)^{-1}(F_o - 1)^{-1}|0 >_R$; the negative sign being due to the fermionic loop. One can proceed a step further and write down the world sheet supersymmetric charge

$$Q = \frac{i}{\pi}\int_0^\pi \rho^0 \rho^{\dagger\alpha} \partial_\alpha X^\mu \Psi_\mu d\sigma \quad (57)$$

and find, as it must,

$$\sum \{Q_\alpha^\dagger, Q_\alpha\} = 2H \qquad \text{and} \qquad \sum_\alpha |Q_\alpha|\phi_o > |^2 = 2 < \phi_o|H|\phi_o > . \quad (58)$$

The ground state is of zero energy. There are no overall tachyons in this Superstring.

To prove the modular invariance, one has to use the G.S.O. condition. In covariant formulation, the number of degrees of freedom of fermions is the number obtained after substraction of constraints from the total number. In our case the total number is 44 and there are four constraints. So the physical fermionic modes is 40. The partition function Z can be found by putting the 8 such fermions(2^3 of SO(6)) in each of five boxes. This is a multiplication of spin structure of eight fermions given by,

$$A_8(\tau) = (\Theta_3(\tau)/\eta(\tau))^4 - (\Theta_2(\tau)/\eta(\tau))^4 - (\Theta_4(\tau)/\eta(\tau))^4 \quad (59)$$

and

$$A_8(1+\tau) = -A(\tau) = A(-\frac{1}{\tau}), \quad (60)$$

where the $\Theta(\tau)$'s are the Jacobi theta function, $q = exp(i\pi\tau)$ and $\eta(\tau)$ is Didekind eta function. Due to the Jacobi relation $A_8(\tau) = 0$, the entire partition function which is the product of all constituent partition functions [14] of the model vanishes.

The zero mass particle spectrum of the quantised action(11) is very large. They are scalars, vectors and tensorial in nature. In reference [12] with Maharana, we have shown that the massless excitations of the standard model can be found from this model. There are also Graviton and Gravitino [18]. For the gauge symmetry, one has to find massless vector bosons which are generators of a group. We follow the work of Li [16].

Consider O(n). There are $\frac{1}{2}n(n-1)$ generators represented by

$$L_{ij} = X_i \frac{\partial}{\partial X_j} - X_j \frac{\partial}{\partial X_i}, \quad i,j = 1, ..., n. \tag{61}$$

Using

$$\left[\frac{\partial}{\partial X_i}, X_j\right] = \delta_{ij}$$

, the Lie algebra which is the commutator relation among the generators

$$[L_{ij}, L_{kl}] = \delta_{jk} L_{il} + \delta_{il} L_{jk} - \delta_{ik} L_{jl} - \delta_{jl} L_{ik}. \tag{62}$$

Hence one must have $\frac{1}{2}n(n-1)$vector gauge bosons W^{μ}_{ij} with the transformation law

$$W^{\mu}_{ij} \rightarrow W^{\mu}_{ij} + \epsilon_{ik} W^{\mu}_{kj} + \epsilon_{jl} W^{\mu}_{li}, \qquad W^{\mu}_{ij} = -W^{\mu}_{ji}, \tag{63}$$

where $\epsilon_{ij} = -\epsilon_{ji}$ are the infinitesimal parameters which characterised such rotation in O(n). Under gauge transformation of second kind,

$$W^{\mu}_{ij} \rightarrow W^{\mu}_{ij} + \epsilon_{ik} W^{\mu}_{kj} + \epsilon_{jl} W^{\mu}_{li} + \frac{1}{g} \partial^{\mu} \epsilon_{ij}. \tag{64}$$

The Yang-Mills Lagrangian is then written as

$$L = -\frac{1}{4} |F^{\mu\nu}_{ij}|^2 \tag{65}$$

with

$$F^{\mu\nu}_{ij} = \partial^{\mu} W^{\nu}_{ij} - \partial^{\nu} W^{\mu}_{ij} + g \left(W^{\mu}_{ik} W^{\nu}_{kj} - W^{\nu}_{ik} W^{\mu}_{kj} \right), \tag{66}$$

$F^{\mu\nu}_{ij}$ has the obvious properties, namely

$$\Box F^{\mu\nu}_{ij} = 0, \qquad \partial_{\mu} F^{\mu\nu}_{ij} = \partial_{\nu} F^{\mu\nu}_{ij} = 0, \qquad F^{\mu\mu}_{ij} = 0. \tag{67}$$

There are two sets of field strength tensors which are found in the model, one for SO(6) and the other for SO(5). Since $\Box F^{\mu\nu}_{ij} = 0$, one can take plane wave solution and write

$$F^{\mu\nu}_{ij}(x) = F^{\mu\nu}_{ij}(p) \; exp(ipx) \tag{68}$$

then

$$p^2 F^{\mu\nu}_{ij}(p) = p_{\mu} F^{\mu\nu}_{ij}(p) = p_{\nu} F^{\mu\nu}_{ij}(p) = F^{\nu\nu}_{ij}(p) = 0, \qquad F^{\mu\nu}_{ij}(p) = -F^{\mu\nu}_{ji}(p). \tag{69}$$

These are physical state conditions (37)-(39) as well.

$$L_0 F^{\mu\nu}_{ij}(p) = 0, \qquad G_{\frac{1}{2}} F^{\mu\nu}_{ij}(p) = 0, \text{and} \qquad L_1 F^{\mu\nu}_{ij}(p) = 0. \tag{70}$$

The field strength tensor is found to be

$$F^{\mu\nu}_{ij}(p) = b^{\mu\dagger}_i \; b^{\nu\dagger}_j |0, p>; \tag{71}$$

(i,j)=1,...,6 for O(6) and 1,....,5 for O(5) with b$'$ replaced by b. For simplicity, we drop the $'$p$'$ dependance and the $\dagger$'s so that $F^{\mu\nu}_{ij} = b^{\mu}_i \; b^{\nu}_j$. In terms of the excitation of the quanta of the string, the vector generators are

$$W^{\mu}_{ij} = \frac{1}{\sqrt{2ng}} \; n_\kappa \epsilon^{\kappa\mu\nu\sigma} b_{\nu,i} b_{\sigma,j} \tag{72}$$

n_κ is the time like four vector and can be taken as (1,0,0,0). One finds that

$$\begin{aligned} \partial^\mu W^{\nu}_{ij} - \partial^\nu W^{\mu}_{ij} &= p^\mu W^{\nu}_{ij} - p^\nu W^{\mu}_{ij} \\ &= \frac{1}{\sqrt{2ng}} \left(n_\kappa \epsilon^{\kappa\nu\lambda\sigma} p^\mu - n_\kappa \epsilon^{\kappa\mu\lambda\sigma} p^\nu \right) b_{\lambda,i} b_{\sigma,j} = 0 \end{aligned} \tag{73}$$

as μ must equal ν if κ, λ, σ are the same. Again,

$$\begin{aligned} g\left(W^{\mu}_{ik} W^{\nu}_{kj} - W^{nu}_{ik} W^{\mu}_{kj} \right) &= \frac{1}{2n} \Big(n_\kappa \; \epsilon^{\kappa\mu\lambda\sigma} \; b_{\lambda,i} b_{\sigma,k} \; n_{\kappa'} \; \epsilon^{\kappa'\nu\lambda'\sigma'} \; b_{\lambda',k} b_{\sigma,j} - \mu \leftrightarrow \nu \Big) \\ &= b^{\mu}_i b^{\nu}_j = F^{\mu\nu}_{ij}. \end{aligned} \tag{74}$$

We have used

$$\{ b_{\lambda' k}, b_{\sigma j} \} = \eta_{\lambda'\sigma} \delta_{kk} = \; n \; \eta_{\lambda'\sigma}.$$

Since the product of pairs of b and b$'$ commute, the gauge group of the action (11) is the product group $SO(6) \otimes SO(5)$. This is same as the symmetry of the action (11).

To descend to the standard model group $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$, it can be normally done by introducing Higgs which breaks gauge symmetry and supersymmetry. However, one uses the method of symmetry breaking by using Wilson's lines, supersymmetry remains in tact but the gauge symmetry is broken. This Wilson loop is

$$U_\gamma = P \; \exp\left(\oint_\gamma A_\mu \; dx^\mu\right). \tag{75}$$

P represents the ordering of each term with respect to the closed path γ. The details of the closed loops are given towards the end of the article for each particular case. SO(6)=SU(4) descends to $SU_C(3) \otimes U_{B-L}(1)$. This breaking can be accomplished by choosing one element of U_0 of SU(4) such that

$$U_0^2 = 1. \tag{76}$$

This element generates the permutation group Z_2. Thus

$$\frac{SO(6)}{Z_2} = SU_C(3) \otimes U_{B-L}(1) \tag{77}$$

without breaking supersymmetry. Similarly $SO(5) \rightarrow SO(3) \otimes SO(2) = SU(2) \otimes U(1)$. We have

$$\frac{SO(5)}{Z_2} = SU(2) \otimes U(1) \tag{78}$$

Thus

$$\frac{SO(6) \otimes SO(5)}{Z_2 \otimes Z_2} = SU_C(3) \otimes U_{B-L}(1) \otimes U_R(1) \otimes SU_L(2) \tag{79}$$

making an identification with the usual low energy phenomenology. But this is not the standard model. We have an additional U(1). There is an instance in E_6 where there is a reduction of rank by one. Following the same idea [4], we may take

$$U_\gamma = (\alpha_\gamma) \otimes \begin{pmatrix} \beta_\gamma & & \\ & \beta_\gamma & \\ & & \beta_\gamma^{-2} \end{pmatrix} \otimes \begin{pmatrix} \delta_\gamma & \\ & \delta_\gamma^{-1} \end{pmatrix} \tag{80}$$

α_γ^3 =1 such that α_γ is the cube root of unity. This structre lowers the rank by one. We have

$$\frac{SO(6) \otimes SO(5)}{Z_3} = SU_C(3) \otimes SU_L(2) \otimes U_Y(1) \tag{81}$$

arriving at the supersymmetric standard model.

We can elaborately discuss the Z_3 described by [13],

$$g(\theta_1, \theta_2, \theta_3) = (\frac{2\pi}{3} - 2\theta_1, \frac{2\pi}{3} + \theta_2, \frac{2\pi}{3} + \theta_3). \tag{82}$$

As promised earlier, for one of the first Wilson loop, the angle integral for $\theta_1 = \frac{2\pi}{9}$ to $\frac{2\pi}{3} - \frac{4\pi}{9} = \frac{2\pi}{9}$, so that the loop integral vanishes. $\theta_2 = 0$ to $2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$ for the second loop described by a length parameter R. $\theta_3 = 0$ to $2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$ for the remaining loop with the same R. We take the polar components of the gauge fields as non zero constants as given below.

$$gA^{15}_{\theta_2} = \vartheta_{15}$$

for SO(6) = SU(4) for which the diagonal generator t_{15} breaks the symmetry and

$$g' A^{10}_{\theta_3} = \vartheta'_{10}$$

for SO(5), the diagonal generator being t'_{10}. The generators of both SO(6) and SO(5) are 4×4 matrices. We can write the Z_3 group as

$$T = T_{\theta_1} T_{\theta_2} T_{\theta_3}. \tag{83}$$

We have T_{θ_1} =1. This leaves the unbroken symmetry $SU(3) \times SU(2)$ untouched

$$T_{\theta_2} = exp\left(i\ t_{15} \int_o^{\frac{4\pi}{3}} \vartheta_{15} R d\theta_2\right) \tag{84}$$

$T_{\theta_2} \neq 1$ breaks the SU(4) symmetry.

$$T_{\theta_3} = exp\left(i\ t'_{10} \int_o^{\frac{4\pi}{3}} \vartheta'_{10} R d\theta_3\right) \tag{85}$$

$T_{\theta_3} \neq 1$ breaks the SO(5) symmetry. But the remaining product of Z_3 is

$$T_{\theta_2} T_{\theta_3} = exp\left(i \int_o^{\frac{4\pi}{3}} (\vartheta'_{10} t'_{10} + \vartheta_{15} t_{15}) R d\theta\right) \tag{86}$$

Since ϑ_{15} and ϑ'_{10} are arbitrary constants, we can choose in such a way that $t_{15}\vartheta_{15} + t'_{10}\vartheta'_{10} = 0, \frac{3}{2R}, ...$ The term in the exponential is zero or multiples of $2\pi i$. Thus T= U(1) and equation (29) is obtained, reducing the rank by one [19].

Thus we have made the successful attempt in constructing a N=1, D=4 superstring with the gauge symmetry $SO(6) \otimes SO(5)$ which, with the help of Wilson lines, descend to the SUSY standard model. Since the Z_3 has three generators and Euler number 3, we get three generations of the standard model because $SO(6) \otimes SO(5) \rightarrow Z_3 \otimes SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$. This is a very important result. The results of this article deserve immediate and serious attention of the physicists interested in String theory.

I have profitted from a discussion with Prof L.Maharana which is thankfully acknowledged.

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